Code No: 133BC

**R16**

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B. Tech II Year I Semester Examinations, September/October - 2023**
**MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE**
**(Common to CSE, IT)**

**Time: 3 Hours** **Max. Marks: 75**

**Note:** i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

## PART – A

**(25 Marks)**

1.a) Express A ↔ B in terms of the connectives {∧, ¬}. [2]

b) Prove that $p \vee (q \wedge r) \leftrightarrow [(p \vee q) \wedge (p \vee r)]$ is a tautology. [3]

c) Suppose R and S are binary relations on a set A. If R and S are reflexive, is R ∩ S reflexive? [2]

d) In a group of 100 students, 72 students can speak English and 43 students can speak Hindi. Based on these data, answer the following:
Find the number of students who can speak both English and Hindi. [3]

e) How many 3-letter words can be formed from the letters of the word ''HAPPY''? [2]

f) In how many ways can 14 people be partitioned into 7 teams where in some order 2 teams have 3 members each, 3 teams have 2 each, and 2 teams have 1 member each? [3]

g) Write a generating function for $a_r$, when $a_r$ is the number of ways of selecting r balls from 3 red balls, 5 blue balls and 7 white balls. [2]

h) Let $a_r$ denotes the number of ways to select r balls from 20 garnet balls, 20 gold balls, 30 green balls, and 30 blue balls with the constraints that the number of garnet balls is not equal to 2, the number of gold balls is not 3, the number of green balls is not 4, and the number of blue balls is not 5. Determine the generating function $a_r$. [3]

i) Give an example of two non-directed graphs with 4 vertices and 2 edges that are not isomorphic. [2]

j) Give an example of a graph which is Hamiltonian but not Eulerian. [3]

## PART – B

**(50 Marks)**

2.a) Show that R → S is logically derived from the premises P → (Q → S), ¬R ∨ P and Q.

b) Obtain the principal conjunctive normal form and principal disjunctive normal form of (¬P → R) ∧ (Q ↔ P) by using equivalences. [5+5]

**OR**

3.a) Show that $(\neg Q \wedge (P \rightarrow Q)) \rightarrow \neg P$.

b) Show that the following premises are inconsistent.
i) If Jack misses many classes through illness, then he fails high school.
ii) If Jack fails high school, then he is uneducated.
iii) If Jack reads a lot of books, then he is not uneducated.
iv) Jack misses many classes through illness and reads a lot of books. [5+5]

4.a) Show that the function $f: Q \to Q$ defined by $f(x) = 2x + 3$ is both one-one and onto. Here, $Q$ is the set of all rational numbers.

b) State and prove Demorgan's law. [5+5]

**OR**

5.a) Prove that in every lattice distributive inequalities are true.

b) If (A, R) is a partially ordered set then show that the set $(A, R^{-1})$ is also a partially ordered set, where $R^{-1} = \{(b, a) / (a, b) \in R\}$. [5+5]

6.a) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7.

b) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (i) they can be male or female, (ii) two must be men and two women and (iii) they must all be of the same sex. [5+5]

**OR**

7.a) Prove that $[C(n,0) + C(n,1) + \cdots + C(n,n)]^2 = C(2n,0) + C(2n,1) + \cdots + C(2n,2n)$.

b) How many arrangements are there of the letters a, b, c, d, e, and f with either a before b, or b before c, or c before d? (By "before", we mean anywhere before, not just immediately before). [5+5]

8.a) In $(1 + X^5 + X^9)^{10}$ find

i) the coefficient of $X^{23}$.

ii) the coefficient of $X^{32}$.

b) Find a recurrence relation for the number of n-digit ternary sequences that have an even number of 0's. [6+4]

**OR**

9.a) Solve the following recurrence relation by substitution.

i) $a_n = a_{n-1} + n$ where $a_0 = 2$

ii) $a_n = a_{n-1} + 3^n$ where $a_0 = 1$.

b) Solve the recurrence relation using generating functions. [5+5]

$a_n - 9a_{n-1} + 20a_{n-2} = 0$ for $n \geq 2$ and $a_0 = -3$, $a_1 = -10$

10.a) Prove that the number of odd degree vertices in any graph is even.

b) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic? [5+5]

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

**OR**

11.a) Give a proof of Euler's formula by using induction on the number of edges.

b) Determine the chromatic number of

i) a bipartite graph,

ii) a wheel with 8 vertices $W_8$,

iii) a complete graph $K_n$,

iv) a cycle graph $C_n$ where n is odd,

v) a cycle graph $C_n$ where n is even. [5+5]

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